Study of LCR Resonant Circuit

Objectives:

(i) To study the behavior of a series LCR resonant circuit and to estimate the resonant frequency and Q-factor.

(ii) To study the behavior of voltage drop across inductor and capacitor and hence estimate the resonant frequency.

Overview:

Circuits containing an inductor *L*, a capacitor *C*, and a resistor *R*, have special characteristics useful in many applications. Their frequency characteristics (impedance, voltage, or current vs. frequency) have a sharp maximum or minimum at certain frequencies. These circuits can hence be used for selecting or rejecting specific frequencies and are also called tuning circuits. These circuits are therefore very important in the operation of television receivers, radio receivers, and transmitters.

Let an **alternating voltage V_i** be applied to an inductor L, a resistor R and a capacitor *C* all in series as shown in the circuit diagram. If *I* is the instantaneous current flowing through the circuit, then the applied voltage is given by

$$V_i = V_{R_{d.c.}} + V_L + V_C = \left[R_{d.c.} + j\left(\omega L - \frac{1}{\omega C}\right)\right] I \qquad \textbf{(1)}$$

Here $R_{d.c.}$ is the total d.c. resistance of the circuit that includes the resistance of the pure resistor, inductor and the internal resistance of the source. This is the case when the resistance of the inductor and source are not negligible as compared to the load resistance R. So, the total impedance is given by

$$Z = \left[R_{d.c.} + j\left(\omega L - \frac{1}{\omega C}\right)\right] \qquad \textbf{(2)}$$

The magnitude and phase of the impedance are given as follows:

$$|Z| = \textbf{(3)}$$

$$\tan\varphi = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R_{d.c.}} \qquad \textbf{(4)}$$

Thus three cases arise from the above equations:

(a) $\omega L > (1/\omega C)$, then tan φ is positive and applied voltage leads current by phase angle φ.

(b) $\omega L < (1/\omega C)$, then tan φ is negative and applied voltage lags current by phase angle φ.

(c) $\omega L = (1/\omega C)$, then $\tan \varphi$ is zero and applied voltage and current are in phase. Here $V_L = V_C$, the circuit offers minimum impedance which is purely resistive. Thus the current flowing in the circuit is maximum (I_0) and also V_R is maximum and V_{LC} (V_L+V_C) is minimum. This condition is known as resonance and the corresponding frequency as resonant frequency (ω_0) expressed as follows:

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{or} f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad \textbf{(5)}$$

At resonant frequency, since the impedance is minimum, hence frequencies near f_0 are passed more readily than the other frequencies by the circuit. Due to this reason LCR-series circuit is called ***acceptor circuit***. The band of frequencies which is allowed to pass readily is called **pass-band**. The band is arbitrarily chosen to be the range of frequencies between which the current is equal to or greater than $I_0/\sqrt{2}$. Let f_1 and f_2 be these limiting values of frequency. Then the width of the band is $BW = f_2 - f_1$.

The **selectivity** of a tuned circuit is its ability to select a signal at the resonant frequency and reject other signals that are close to this frequency. A measure of the selectivity is the **quality factor** (**Q**), which is defined as follows:

$$Q = \frac{f_0}{f_2 - f_1} = \frac{\omega_0 L}{R_{d.c.}} = \frac{1}{R_{d.c.}\omega_0 C} \qquad \textbf{(6)}$$

In this experiment, you will measure the magnitude and phase of V_R and V_{LC} with respect to V_i ($|(V_R/V_i)|$, $|(V_{LC}/V_i)|$, Φ_R and Φ_{LC} in the vicinity of resonance using following working formulae.

$$\left|\frac{V_R}{V_i}\right| = \frac{R}{|Z|} \qquad \textbf{(7)} \qquad \varphi_R = -\tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R_{d.c.}}\right) \qquad \textbf{(8)}$$

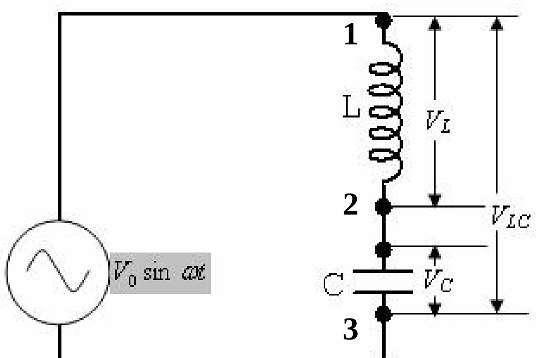
and

$$\left|\frac{V_{LC}}{V_i}\right| = \frac{\omega L - \frac{1}{\omega C}}{|Z|} \qquad \textbf{(9)} \qquad \varphi_{LC} = \tan^{-1}\left(\frac{R_{d.c.}}{\omega L - \frac{1}{\omega C}}\right) \textbf{(10)}$$

Circuit Components/Instruments:

(i) Inductor, (ii) Capacitor, (iii) Resistors, (iv) Function generator, (v) Oscilloscope, (vi) Multimeter/LCR meter, (vii) Connecting wires, (viii) Breadboard

Circuit Diagram:



Procedure:

(I) Measuring V_R, V_{LC} and Φ_R, Φ_{LC}:

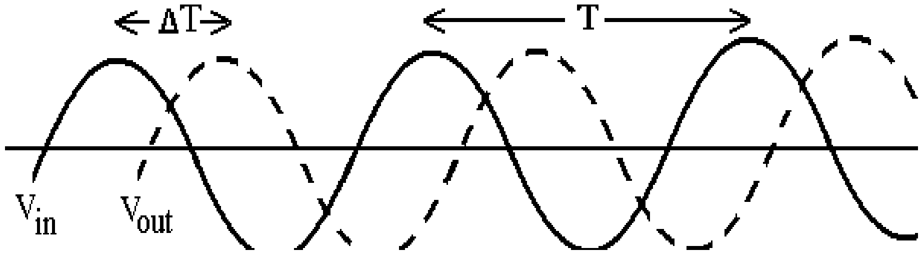
(a) Using the multimeter/LCR meter, note down all the measured values of the inductance, capacitance and resistance of the components provided. Also, measure the resistance of the inductor. Calculate the d.c. resistance of the circuit. Calculate the resonant frequency.

(b) Configure the circuit on a breadboard as shown in circuit diagram. Set the function generator **Range** in 20 KHz and **Function** in sinusoidal mode. Set an input voltage of 5V (peak-to-peak) with the oscilloscope probes set in X1 position. Set the function generator probe in X1 position.

(c) Feed terminals 1,4 in the circuit diagram to channel 1 and 3,4 to channel 2 of the oscilloscope to measure input voltage V_i and output voltage V_R, respectively. Note that terminal 4 is connected to the ground pin of the function generator and oscilloscope.

(d) Vary the frequency in the set region slowly and record V_R and V_i (which may not remain constant at the set value, guess why?). Read the frequency from oscilloscope.

For each listed frequency, also measure the phase shift angle Φ_R with proper sign directly from oscilloscope. You could also calculate phase shift by measuring lead/lag time, ΔT, as shown in the diagram below using the expression, $\varphi_R(\text{deg}) = \left(\frac{\Delta T}{T}\right) \times 360^0$.



(e) Replace the resistor with another value and repeat steps (c) and (d). No phase measurement is required.

(f) Now, interchange the probes of the function generator and oscilloscope, i.e. make terminal 1 as the common ground so that you will measure V_{LC} output between terminal 3 and 1 and V_i between 4 and 1. Repeat step-(d) to record V_{LC}, V_i and Φ_{LC}.

(II) Measuring V_L and V_C:

(a) Go back to the original circuit configuration you started with. Interchange R with L to measure V_i and V_L (see steps (c) and (d) of the previous procedure). Calculate V_L/V_i for each frequency.

(b) Now, interchange the inductor with capacitor and measure V_i and V_C. Calculate V_C/V_i for each frequency.

Observations:

L = _________ mH, C = ________ μF, $f_0 = \frac{1}{2\pi\sqrt{LC}}$ = _________ **kHz**

Internal resistance of inductor = _________ Ω

Output impedance of Function generator = __________ Ω

Table:1 **R_1 = _______ Ω**

Sl.No.	**f (kHz)**	**V_i (V)**	**V_R (V)**	**V_R/V_i**	**V_R/V_i (Calculated)**	**Φ_R**	**Φ_R (Calculated)**

Table:2 **R_2 = _______ Ω**

Sl.No.	**Frequency, f (kHz)**	**V_i (V)**	**V_R (V)**	**V_R/V_i**	**V_R/V_i (Calculated)**

Table:3 **R_1 = _______ Ω**

Sl.No.	Frequency, f (kHz)	V_i (V)	V_{LC} (V)	V_{LC}/V_i	V_{LC}/V_i (Calcu-lated)	Φ_{LC} (deg)	Φ_{LC} (deg) (Calculated)

Graphs:

(a) Plot the observed values of V_R/V_i, V_{LC}/V_i, Φ_R and Φ_{LC} versus frequency. Estimate the resonant frequency.

(b) Plot V_R/V_i versus frequency for both the resistors on the same graph-sheet and compare their behavior. Estimate the Q-factor in each case and compare with calculated values.

Discussions/Results:

Precautions: Make the ground connections carefully.